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Cloud Travel Slide Rules for the Determination of Downwind Travel and Area Coverage

by

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ARMY CHEMICAL CENTER, MD.

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FOREWORD

The work described herein was authorized under Task 4C04-15-029-09, Mathematical Research on Dissemination (U). The work was started in September 1958 and completed September 1961.

Notice

Disposition

DIGEST

Slide rules have been contrived for evaluating O. G. Sutton's mathematical models for estimating the downwind travel and area coverage from an instantaneous point source of vapor or very fine aerosol disseminated and sampled at ground level.

Using these slide rules, the nonmathematician can employ the mathematical theory for making preliminary weapons-effects predictions necessary in handling everyday problems encountered in research and development of chemical-weapons systems.

CLOUD TRAVEL SLIDE RULES FOR THE DETERMINATION OF DOWNWIND TRAVEL AND AREA COVERAGE

I. INTRODUCTION.

In the research and development of a chemical-weapons systems, determination of the weapons characteristics for producing the optimum weapons effects is basic. The large number of meteorological and physical conditions to be considered in a weapons-evaluation study precludes field experimentation as an adequate technique for evaluation. Thus, mathematical models have been developed to estimate expected results; however, the mathematical forms of these models are such that considerable numerical manipulations are required in determining a set of data. Thus, the mathematical models for estimating downwind cloud travel and area coverage have been reduced to the form of a mechanical slide rule in order to simplify sufficiently some aspects of weapons-effects predictions in such a manner that the nonmathematician can apply the theory to everyday problems encountered in research and development of chemical-weapons systems.

II. THEORY.

A. Diffusion Equations.

Sutton's* nonisotropic diffusion equation for predicting total dosage downwind from an instantaneous point source composed of vapor or very fine aerosols disseminated at ground level is

$$D(x,y,z) = \frac{2Q}{\pi C_y C_z \bar{u} x^{2-n}} \left[\exp \left(\frac{-y^2}{C_y^2 x^{2-n}} \right) \right] \left[\exp \left(\frac{-z^2}{C_z^2 x^{2-n}} \right) \right] \quad (1)$$

where

D = dosage, mg min/cu m

Q = total mass of agent dispersed, mg

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* O. G. Sutton. The Problem of Diffusion in the Lower Atmosphere. Quart. J. Roy. Meteorol. Soc. 73, 268 (1947).

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$\left.\begin{matrix} C_y \\ C_z \end{matrix}\right\}$ = generalized coefficients of diffusion crosswind and vertically, $m^{\frac{n}{2}}$

$\bar{u}$ = mean wind speed, m/min

x = distance downwind from source, m

y = distance crosswind from mean wind track or x-axis, m

z = sam 'ng height above ground, m

n = parameter related to atmospheric stability

It is seen from equation (1) that the ground-level dosage is given by

$$D(x, y, o) = \frac{2Q}{\pi C_y C_z \bar{u} x^{2-n}} \exp\left(\frac{-y^2}{C_y^2 x^{2-n}}\right) \tag{2}$$

The expression describing the contour of dosage D is found by rearranging equation (2)

$$y = C_y x^{\frac{(2-n)}{2}} \left(\ln \frac{a}{x^{2-n}}\right)^{\frac{1}{2}} \tag{3}$$

where

$$a = \frac{2Q}{\pi C_y C_z \bar{u} D(x, y, o)}$$

Since $y = o$ at $x = o$ and at $x = a^{\frac{1}{2}(2-n)}$, Sutton's expression for the area enclosed by this contour is given by

$$A = 2 \int_o^{a^{\frac{1}{(2-n)}}} y dx = \frac{\sqrt{\pi} C_y \left[\frac{2Q}{\pi C_y C_z \bar{u} D(x, y, o)} \right]^{\frac{(4-n)}{(4-2n)}}}{(2-n) \left(\frac{4-n}{4-2n} \right)^{\frac{3}{2}}} \tag{4}$$

From equation (2), it is seen that the ground-level dosage along the mean wind track is

$$D(x, o, o) = \frac{2Q}{\pi C_y C_z \bar{u} x^{2-n}} \tag{5}$$

The ground-level semiwidth, y, of the cloud at axial distance x downwind from the source is the distance from the x-axis at which the dosage, D(x, y), is the fraction K of the axial value, D(x, o). With equations (2) and (5), it is easily deduced that

$$y = C_y x^{\frac{(2-n)}{2}} \sqrt{-\ln K} \tag{6}$$

B. Diffusion Coefficients.

Sutton's* definitive expressions for the diffusion coefficients C_z and C_y for airflow over an aerodynamically smooth surface are

$$\left[C_z^2 ; C_y^2 \right] = \frac{4\nu^n}{(1-n)(2-n)\bar{u}^n} \left[\left(\frac{\overline{w'^2}}{\bar{u}^2} \right)^{1-n} ; \left(\frac{\overline{v'^2}}{\bar{u}^2} \right)^{1-n} \right] \tag{7}$$

* O. G. Sutton. The Theoretical Distribution of Airborne Pollution From Factory Chimneys. Quart. J. Roy. Meteorol. Soc. 73, 426-436 (1947).

where

ν = kinematic viscosity of air (equal to 0.143 x 10^{-4} sq m/sec at 1,000 mb and 10°C)

$\bar{u}$ = mean wind speed, m/sec

$\bar{w}'$, $\bar{v}'$ = eddy velocities in vertical and crosswind, respectively.

n = a dimensionless stability parameter.

Sutton assigned values of n, $\bar{u}$, and the ratios $(\bar{w}'/\bar{u})_n/(\bar{w}'/\bar{u})_{\frac{1}{4}}$ as being representative for each of four stability conditions shown in table 1.

TABLE 1

VALUES OF n, $\bar{u}$, AND $\bar{w}'/\bar{u}$ FOR VARIOUS STABILITY CONDITIONS (Sutton)

Stability condition	n	$\bar{u}$	$(\bar{w}'/\bar{u})_n/(\bar{w}'/\bar{u})_{\frac{1}{4}}$
		m/sec	
Large lapse	1/5	7	2
Neutral	1/4	5	1
Moderate inversion	1/3	3	1/2
Large inversion	1/2	2	1/4

For practical purposes, the following (table 2) may be used as a guide for defining stability conditions.

Sutton has also assigned values for C_z (table 3) at various heights as being appropriate for neutral conditions in a wind of 5 m/sec over smooth terrain.

TABLE 2

STABILITY PARAMETERS

Temperature gradient $T_{4.0\ m} - T_{0.5\ m}$	Stability conditions	n
°F		
$\Delta T < -0.4$	Lapse	0.20
$-0.4 \leq \Delta T \leq 0.4$	Neutral	0.25
$0.4 < \Delta T \leq 2.0$	Moderate inversion	0.33
$\Delta T > 2.0$	Large inversion	0.50

TABLE 3

VALUES OF C_z FOR VARIOUS HEIGHTS WHEN $n = \frac{1}{4}$, $\overline{u} = 5$ m/sec

(Sutton)

Height	C_z
m	$m^{\frac{n}{2}}$
0-25	0.12
50	0.10
75	0.09
100	0.07

Sutton* assumes $C_y = C_z$ at a height of 25 m and above. In order to obtain values of C_y for the various stability conditions at heights <25 m, however, values of the ratios $(\overline{v}'/\overline{u}')_n/(\overline{v}'/\overline{u})_{\frac{1}{4}}$ are needed. Best** concluded, from data collected from extensive tests, that the ratio of lateral to vertical gustiness, $\overline{v}'/\overline{w}'$, is 1.81 irrespective of wind velocity and temperature gradient. Using this result with equation (7), the following relationship is obtained:

$$C_y = (1.81)^{1-n} C_z \tag{8}$$

With equations (7) and (8) and tables 1 and 3, values of the diffusion coefficients may be determined for the appropriate conditions as listed and specified by Sutton.

By assuming that the ratios $(\overline{w}'/\overline{u})$ and $(\overline{v}'/\overline{u})$ are constant for various values of $\overline{u}$ (Sutton† says they are "practically independent of the magnitude of $\overline{u}$."), extensions of these coefficients to include other wind speeds may easily be accomplished. For constant n, it may be deduced from equation (21) that

$$\left[C_{z_2};\ C_{y_2}\right] = \left[C_{z_1};\ C_{y_1}\right] \left(\frac{\overline{u}_1}{\overline{u}_2}\right)^{\frac{n}{2}} \tag{9}$$

The above outlined procedure was used to determine the diffusion coefficients used here. A more detailed explanation of the procedure along with tabulated parameter values for a variety of conditions are contained in a previously published report.‡

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* O. G. Sutton. The Theoretical Distribution of Airborne Pollution From Factory Chimneys. Quart. J. Roy. Meteorol. Soc. 73, 426-436 (1947).

** A. C. Best. Transfer of Heat and Momentum in the Lowest Layers of the Atmosphere. Geophys. Memoirs No. 65. Air Ministry Meteorol. Office. London. 1935.

† O. G. Sutton. The Problem of Diffusion in the Lower Atmosphere. Quart. J. Roy. Meteorol. Soc. 73, 268 (1947).

‡ D. O. Egner et al. CRDLR 3002. A Preliminary Theoretical Study of the Cloud Travel of Aerosol Particles Having Diameters Between 20 and 180 Microns. July 1960.

III. DESCRIPTION AND USE OF CLOUD TRAVEL SLIDE RULES.

A. Dosage Field Slide Rule.

The dosage field slide rule is used to determine total dosage at any point, x, y, downwind from an instantaneous point source disseminated at ground level over smooth terrain under various wind speeds and atmospheric stability conditions. The front side of the slide rule is used to determine the total dosage along the mean wind track, D(x, o), and the reverse side is used to determine the semiwidth, y, of the cloud, containing a desired dosage at a specified downwind distance. Thus, the entire dosage field may be established by determining the dosage values at a sufficient number of points. The scales* for the dosage field slide rule are shown in figure 1.

The following two examples illustrate the use of both sides of this slide rule. Given any four parameters included on a side, the fifth parameter value may be obtained by rearrangement of the steps outlined in the examples.

PROBLEM 1: Find the ground level total dosage along the mean wind track at 100 m downwind from a 1,000 gm instantaneous point source disseminated at ground level under the following conditions: Wind speed at 2 m height = 2 m/sec, neutral stability (n = 0.25).

SOLUTION:

1. Use front side of dosage field slide rule.
2. Set hairline of cursor on Q value of 1000 on D scale.
3. Move center slide until assumed stability parameter, n, of 0.25 on cursor intercepts assumed wind speed, $\overline{u}$, of 2 m/sec on C scale.
4. Move cursor until n value of 0.25 on cursor intercepts desired downwind distance, x, of 100 m on B scale.
5. Read dosage on A scale indicated by hairline of cursor (D = 65 mg min/cu m).

PROBLEM 2: For the same source strength, wind speed, and stability condition assumed in problem 1, find the semiwidth of the cloud, y, containing a dosage of 6.5 mg min/cu m, at a downwind distance of 100 m.

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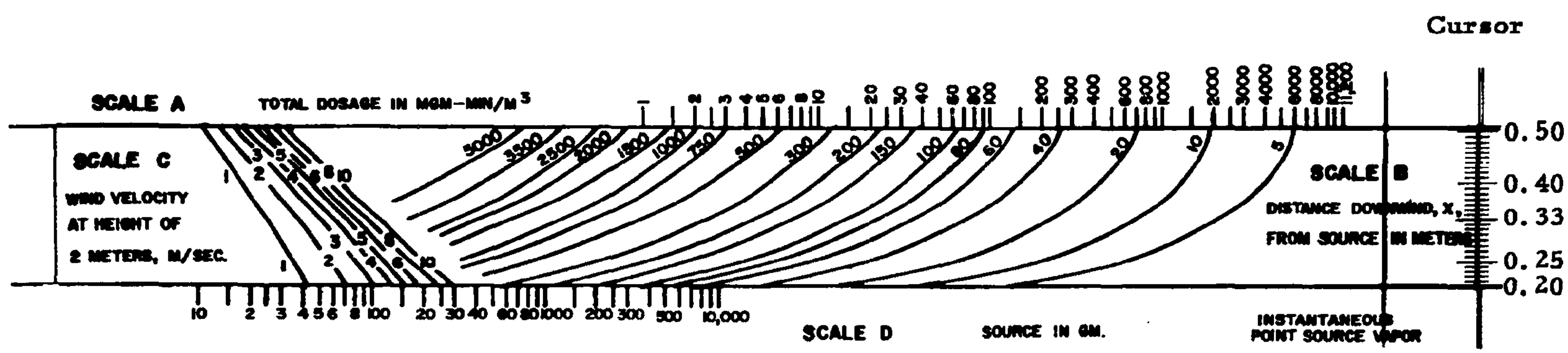
* The unlabeled values on the cursors are values of the stability parameter, n, as given in table 2.

FRONT



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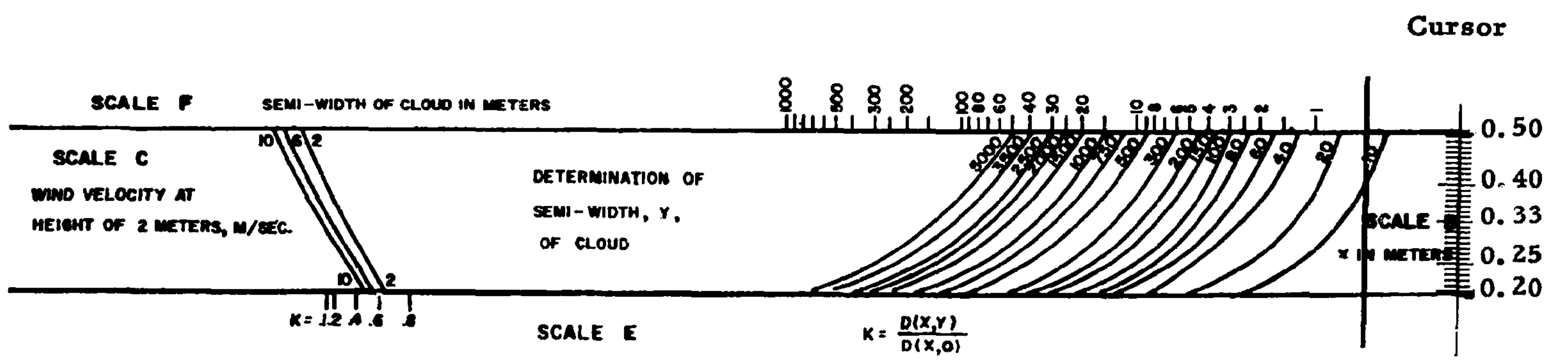


FIGURE 1

DOSAGE FIELD SLIDE RULE

SOLUTION: 1. Use back of dosage field slide rule.

2. In problem 1, the center-line dosage at 100 m downwind was found to be 65 mg min/cu m. Thus,

$$K = \frac{D(x, y)}{D(x, o)} = \frac{6.5}{65} = 0.1$$

3. Set hairline of cursor at K = 0.1 on E scale.

4. Move center slide until assumed stability parameter, n, of 0.25 on cursor intercepts assumed wind speed, $\overline{u}$, of 2 m/sec on C scale.

5. Move cursor until n value of 0.25 on cursor intercepts desired downwind distance, x, of 100 m on B scale.

6. Read semiwidth of cloud on F scale indicated by hairline of cursor (y = 17 m).

B. Area Dosage Slide Rule.

The front side of the area dosage slide rule is used to determine expected area coverage to a desired dosage from an instantaneous point source under the same range of wind speeds and stability conditions considered by the dosage field slide rule. The reverse side is an exact duplicate of the front side of the dosage field slide rule and requires no further explanation. The scales* for the area dosage slide rule are shown in figure 2.

The following example illustrates the use of this slide rule for determining area coverage. As in the instance of the dosage field slide rule, given any four of the parameters included, the fifth parameter value may be obtained by rearrangement of the steps outlined in this example.

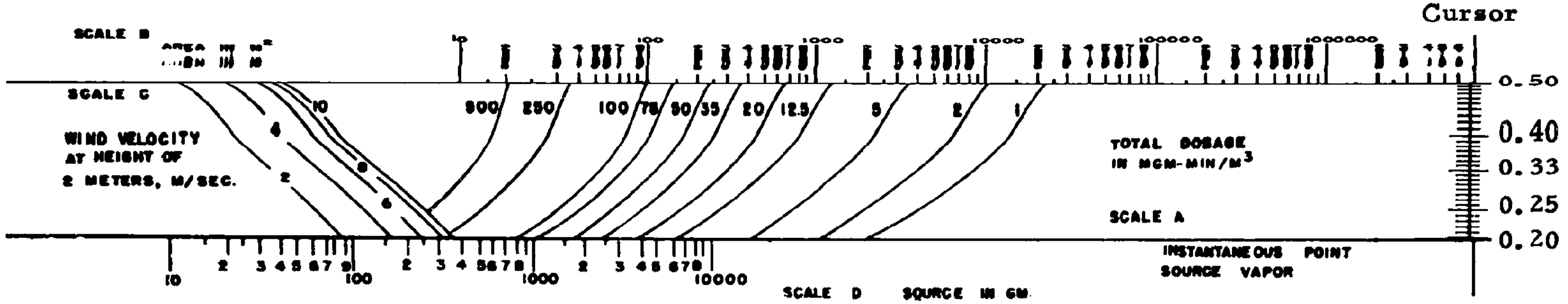
PROBLEM: Find the area covered to a dosage of 100 $\frac{\text{mg min}}{\text{cu m}}$ from a 1,000 gm instantaneous point source disseminated at ground level under the following conditions:

Wind speed at 2 m height = 2 m/sec

Neutral stability (n = 0.25)

* The unlabeled values on the cursors are values of the stability parameter, n, as given in table 2.

FRONT



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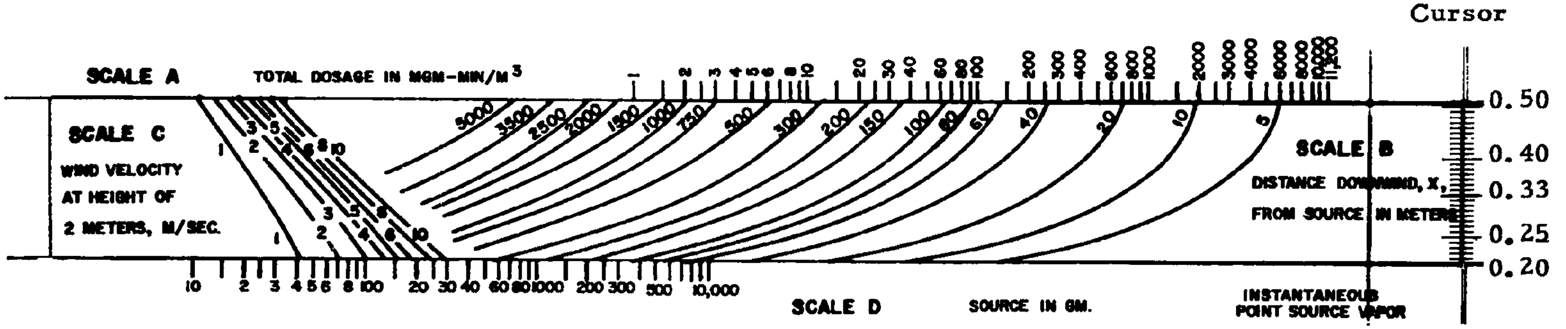

FIGURE 2

AREA DOSAGE SLIDE RULE

SOLUTION: 1. Set hairline of cursor on Q value of 1000 on D scale.

2. Move center slide until assumed stability parameter, n, of 0.25 on cursor intercepts assumed wind speed, $\bar{u}$, of 2 m/sec on C scale.

3. Move cursor until n value of 0.25 on cursor intercepts desired dosage of 100 $\frac{\text{mg min}}{\text{cu m}}$ on A scale.

4. Read area on B scale indicated by hairline of cursor (A = 610 sq m).

IV. CONCLUSIONS.

Slide rules have been contrived for evaluating O. G. Sutton's mathematical models for estimating the downwind travel and area coverage from an instantaneous point source of vapor or very fine aerosol disseminated and sampled at ground level.

Using these slide rules, the nonmathematician can employ the mathematical theory for making preliminary weapons-effects predictions necessary in handling everyday problems encountered in research and development of chemical-weapons systems.

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1. Vapor Cloud Travel Model
2. Vapor Slide Rules
3. Area Coverage
4. Sutton's Model

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